

Exercise 10

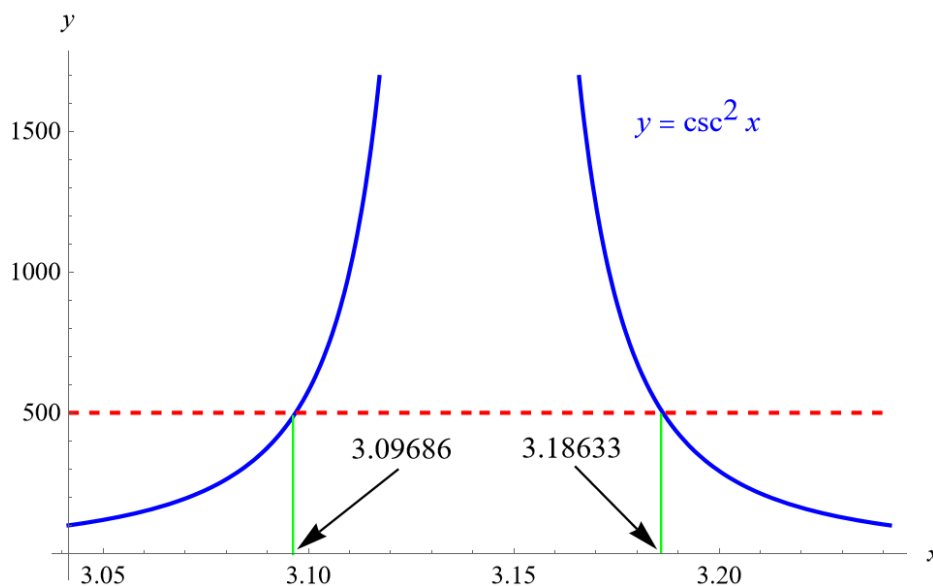
Given that $\lim_{x \rightarrow \pi} \csc^2 x = \infty$, illustrate Definition 6 by finding values of δ that correspond to (a) $M = 500$ and (b) $M = 1000$.

Solution

For $M = 500$, Definition 6 says that this limit is equivalent to

$$\text{if } 0 < |x - \pi| < \delta \quad \text{then} \quad \csc^2 x > 500$$

for some positive δ .

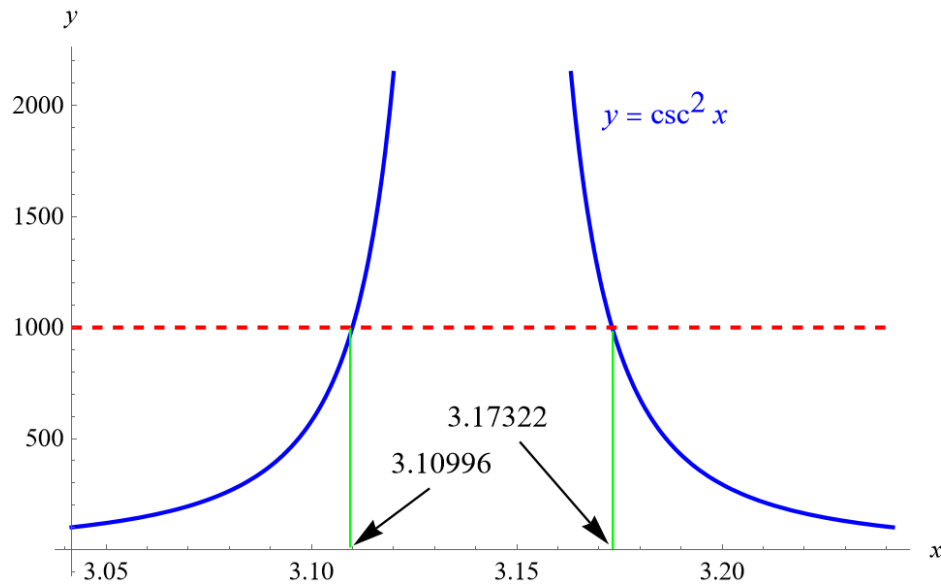


As long as δ is less than about $3.18633 - \pi \approx 0.0447373$, the function is greater than 500.

For $M = 1000$, Definition 6 says that this limit is equivalent to

$$\text{if } 0 < |x - \pi| < \delta \quad \text{then} \quad \csc^2 x > 1000$$

for some positive δ .



As long as δ is less than about $3.17322 - \pi \approx 0.0316273$, the function is greater than 1000.